Variance of some information measures for truncated random variables with applications

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Abstract:

Information measures are fundamental tools in the field of information theory, enabling us to quantify the amount of information, uncertainty, or divergence within a given system. These measures provide insights onto the outcomes of a random process, and the differences between distributions, enabling a deeper understanding of information processing and transmission. Among the key concepts of information measures are Shannon entropy, Kerridge inaccuracy, and Kullback-Leibler (K-L) divergence. All these measures offer different perspectives on the amount of information on probability distributions. In reliability theory and survival analysis, the concepts of residual, past, and interval lifetimes are crucial for understanding and quantifying the dynamics of uncertainty over time. When incorporated into the above information measures, it provides valuable insights into the behaviour of systems over time. Entropy, inaccuracy, and K-L divergence mathematically represent the average amount of information about the outcomes carried by some baseline distribution. Thus, they are not that effective in the sense that the exact findings may be quite different from the anticipated one if the process of evaluation is repeated. This is because they are calculated as the expected value of information. It would be useful to study some notion that measures the concentration of information around these expected values. The study of variance of information measures is an important development in this regard.

Chapter 1 is introductory, while **Chapter 2** introduces a stochastic order based on residual varentropy, useful to compare heavy tailed distributions in dynamic setup. Further, its relation to the residual entropy order and the usual stochastic orders is explored. Some theoretical results pertaining to the order under transformations are discussed, analyzing the additional conditions. Also, ageing classes based on residual varentropy are defined in this chapter, following some usefulness of the proposed order in the identification of a better system.

In **Chapter 3**, we define a dual stochastic order for varentropy in past lifetime, i.e., varentropy order based on past lifetime. Following Chapter 2, we investigate its relation with the past entropy order and existing stochastic orders. The effect of transformation on the defined order is explored, and new ageing classes based on past varentropy are provided. The efficiency of this order is illustrated in model selection, particularly when the entropy method fails.

Chapter 4 defines measure of varentropy for a doubly truncated random variable called interval varentropy. A detailed theoretical analysis has been carried out to enhance the doubly

truncated entropy measure. Further, the interval varentropy has been studied under monotonic transformations in addition to bounds. A parametric estimator of the measure is proposed and a simulation study is carried out to support our intuition of decreasing varentropy for shrinking intervals and has been applied to two real data sets. Moreover, applications of the defined measure are given related to the choice of most acceptable system and first-passage times of an Ornstein-Uhlenbeck jump-diffusion process.

In **Chapter 5**, we introduce dynamic varinaccuracy measures, precisely for residual and past lifetimes. The study is aimed at enhancing the results of varentropy for residual and past lifetimes. Theoretical results analyzing the measure over time, considering several significant distributions are presented in a detailed manner. Further, the measures are studied for proportional hazard rate and proportional reversed hazard rate models and their relation with the corresponding varentropy measures is given. Some characterization results related to residual varinaccuracy are also presented. Moreover, asymptotic results have been obtained along with application in model selection.

Chapter 6 aims to propose a dispersion measure based on K-L divergence measure for residual lifetimes. Various properties of the measure, such as its rate of change with time, effect under affine transformation, and strictly monotone transformation, have been discussed. Several illustrations are given as applications of the study. A simulation study is also performed to analyze the defined measure between a proposed smooth kernel based non-parametric estimator and a parametric estimator of a given distribution. A dispersion measure for its dual setup, i.e., for past lifetime, has also been introduced and studied. Further, some applications of the proposed measures are presented, including reliability modeling.

Key Words and Phrases: Dispersion, divergence, entropy, inaccuracy, truncated lifetime, varentropy, varinaccuracy.